Assessment of Current Density Singularity in Electromigration of Solder Bumps

Pridhvi Dandu and Xuejun Fan
Department of Mechanical Engineering
Lamar University
PO Box 10028, Beaumont, TX 77710, USA
Tel: 409-880-7792; xuejun.fan@lamar.edu

Abstract

This paper investigates the current density singularity in electromigration of solder bumps. A theoretical analysis is performed on a homogenous wedge with arbitrary apex angle, 2(“ –‸), when the current flow passes through. A potential difference is applied at a distance far away from the tip of the wedge. It is found that current density singularity exists at the tip of the wedges when the angles –‸ ωπ°. The acute angles represent the corner configuration of the actual solder bump and the interconnect. The current crowding in bumps is a result of singularity exhibited at such corners. Finite element results confirm that the maximum current density has strong dependence on mesh size. To eliminate the singularity effect, a volume-averaged current density approach, over a crescent shape where the maximum current density occurs, is suggested. Such an averaged value represents the concentrati02 Tsj,36.7( )6f(r)6.2(j,36.curre)5.2(nt) i02 Tw(n)-8.2nfinntn,36.d 5(i)4.2(s)7.88(ai)18.sois

where ρ is resistivity of the material. The boundary conditions (BC) required on the faces of wedge are:

$$J_\phi = 0 \text{ at } \phi = \pm\phi_0 \text{ for all } r \tag{5}$$

Substituting the BC, $J_\phi = 0$ in the Equation (4) leads to

$$\frac{\partial V}{\partial \phi} = -\frac{1}{\rho}\frac{1}{r} r^{\lambda} F'(\phi) = 0 \quad \text{at } \phi = \pm\phi_0 \text{ for all } r \tag{6}$$

Since $\lambda \neq 0$, at $\phi = \pm\phi_0$ we have

$$F'(\phi) = 0 \tag{7}$$

From the governing equation of electrostatics, we have

$$\nabla^2 V = 0 \tag{8}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}$, substituting the assumed voltage function, into the Equation (8), we get a homogeneous linear differential equation with constant coefficients as shown

$$F''(\phi) + \lambda^2 F(\phi) = 0 \tag{9}$$

The solution for $F(\phi)$, obtained by solving the above equation is

$$F(\phi) = A\cos\lambda\phi + B\sin\lambda\phi \tag{10}$$

where the constants *A* and *B* are to be determined. Applying Equation (7) to the wedge faces which are at angles ($\phi = \phi_0$) and $\phi = -\phi_0$, to the general form of $F(\phi)$ in Equation (10) produces a system of two simultaneous equations with unknown constants. These equations can be represented in a matrix form as:

$$\begin{bmatrix} -\sin\lambda\phi_0 & \cos\lambda\phi_0 \\ \sin\lambda\phi_0 & \cos\lambda\phi_0 \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = 0 \tag{11}$$

Since Equation (11) is homogeneous, the determinant of coefficient matrix must be equal to zero in order to get meaningful solutions. So,

$$\sin 2\lambda\phi_0 = 0 \tag{12}$$

and, the roots of this equation are obtained as follows,

$$\lambda = \frac{n\pi}{2\phi_0} \quad \text{where } n = 0, 1, 2, \ldots \tag{13}$$

Substituting the value of λ in the voltage function, we have the general solution for a wedge with arbitrary apex angle, $2(\phi = \phi_0)$:

$$V(r,\phi) = \ldots r^{\lambda}$$

For – [illegible] ! ÃÏÓ [illegible] " ÓÉÎ [illegible] ρω

In Equations (18) and (19) the exponential of *r* is negative for both the angles. When *r* approaches zero, [illegible] approaches infinity. Subsequently current density also equals to infinity.

Figure 7 Three different mesh schemes

To remove the singularity effect, one method is to extract the averaged current density over certain volumes. In this method the current density is averaged over all the elements of the selected volumes as following:

$$\text{Ўů}_{\text{ﺰﻄﺬ}} \quad \frac{\Sigma \text{Ўů} \;\text{ℷ}\; \text{ώ}_{\text{ﻢ}}}{\Sigma \text{ώ}_{\text{ﻢ}}} \qquad \text{ςς}$$

where, $\text{Ўů}_{\text{ﺰﻄﺬ}}$ is average current density, Ўů is current density in each element, $\text{ώ}_{\text{ﻢ}}$ is volume of each element. Two different volume averaging approaches are studied. In the first mesh sizes are tabulated in Table 1. As the mesh size is reduced by a factor of four, the maximum current density has increased by 24%. The average current densities calculated in the bottom disk for all the mesh schemes are the same. It is observed that the averaging has decreased the current density value in bottom disk by 56% compared to the maximum current density (in the case of ‘X/4’ mesh size).

Figure 12 Current density contours with different meshes in the bottom disk

Table 1 The Maximum and Average Current Densities Calculated in Bottom Disk for Different Mesh Sizes

Mesh Size	Max. Current dens. (Bottom Disk & Crescent) (A/m^2)	Average Current dens. in Bottom Disk (A/m^2)
x	0.55e8	0.30e8
x/2	0.60e8	0.30e8
x/4	0.69e8	0.30e8

Table 2 summarizes all the averaged current density values calculated for all thicknesses (p/r = 0.2, 0.4 and 0.6) of crescents with differen